Maths for Computing
Tutorial 10

1. Let G be a graph of n vertices. Prove that if $degree(u) + degree(v) \geq n - 1$ for every two nonadjacent vertices u and v of G, then G is connected.

2. Prove that if a graph G contains a walk from u to v of length at most l, then it also contains a path from u to v of length at most l.

3. Let $\delta(G) = \{d(v) \mid v \in V\}$ denote the minimum degree of $G = (V, E)$. Prove that G contains a path of length $\delta(G)$ and a cycle of length at least $\delta(G) + 1$.

4. A simple graph is called regular if all its vertices have the same degree. Let G be a connected regular graph with 22 edges. How many vertices can G have?

5. Let G be graph on 10 vertices and 28 edges. Prove that G contains a cycle of length 4.

6. Let G be graph of 3 or more vertices. Then G is connected if and only if G contains two distinct vertices u and v such that $G - u$ and $G - v$ are connected.

7. Two graphs have the same ordered degree sequence. Show that they are not necessarily isomorphic.